Path following control for the Cartesian position of the Quadrotor *

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Abstract: In this paper, we propose a path following controller for the Cartesian position of the quadrotor mobile robot. The control approach is based on a combination of a backstepping inspired controller and an output maneuvering controller, as introduced in (R. Skjetne and Kokotovic (2004)). It is shown that the closed-loop system is locally asymptotically stable and the path following error converges asymptotically to zero when an accurate quadrotor localization system is available, and it is shown that when the quadrotor position is not accurately estimated (for instance, under the influence of noise measurements) the closed-loop dynamics remains bounded. Numerical simulations are performed to show the overall performance of the proposed scheme in both cases.

Keywords: Quadrotor, Nonlinear control, Path following.

1. INTRODUCTION

In recent years, autonomous aerial robots have demonstrated their usefulness in different applications ranging from very expensive military applications to relatively cheap and mass produced home applications. To increase their civilian applications, for instance in surveillance applications, their capability to plan paths and to follow them accurately, specially when non expensive position sensors are available, is of great importance.

Autonomous robot localization is a fundamental problem in autonomous robot applications, the many partial solutions can roughly be categorized into two groups: relative and absolute position measurements, (J. Borenstein and Wehe (1997)). For indoor applications both solutions, in particular odometry, inertial navigation, active beacons and landmark navigation, provide relatively accurate position measurements mainly because the environment is in some sense controlled. For instance, the terrain can be carefully selected to reduce slipping for a successful odometry application. However, for outdoor applications the autonomous robot localization problem has not many successful solutions; since it is difficult to control the environment. Some promising solutions for autonomous robot localization have been obtained by fusing global positioning systems (GPS), inertial measurement units (IMU) and compass sensors (CS) signals. However, due to the deliberated small errors in timing and satellite position of the GPS, there is a bias error associated to the GPS, IMU and CS based position measurement.

Current position measurement systems available in the civil market, based on GPS, IMU and MC, have an associated bias of approximately 5 m virtually unchanging over a long period of time and a random error in the range of 2 – 3 m. A successful control strategy to perform trajectory tracking or path following must be robust against this position measurement errors. For instance, in (D. R. Nelson and Beard (2007)) a successful path following scheme for miniature air vehicles is achieved in the presence of this kind of localization errors. Even though, the closed-loop stability analysis in the presence of localization errors is not performed, the experimental results show that the controller is able to handle such localization errors.

Control strategies for trajectory tracking have the objective of driving the mobile robot at a certain point of the trajectory at a particular time. Hence, trajectory tracking control strategies rely on precise mobile robot localization algorithms. The trajectory tracking error will increase without taking into account the actual position of the mobile robot. Trajectory tracking controllers for the quadrotor has been achieved using control schemes ranging from PID control (G. M. Hoffman and Tomlin (2008)) to nonlinear designs such as feedback linearization (Mokhatari and Benallegue (2004)), backstepping (T. Hamel T. and Ostrowski (2002)), (S. A. Araujo-Estrada and Rodríguez-Cortés (2009)), (N. Gueudard and Mahony (2008)), nested saturations (P. Castillo and Dzul (2005)), Lyapunov based output feedback schemes (L. DongBin and Dawson (2007)) and nonlinear $\mathcal{H}_\infty$ control (G. V. Raffo and Rubio (2010)). On the other hand, control strategies for path following have the primary objective of driving the mobile robot to the path unrelated to time and as a secondary objective to satisfy an additional dynamic specification, for instance the speed along the path.

The rest of the paper is organized as follows. In Section 2 we describe the dynamic model of the quadrotor mobile robot and formulate the path following control problem. Section 3 presents a nonlinear control law to solve the path following problem and discusses the stability of the result-
The vehicle dynamics in the body frame is described by the center of mass of the aircraft structure (body frame) $x$ into the earth and 0 inertial frame (earth frame) such that $z$ points downwards into the earth and $0^b y^b z^b$ a right-hand frame fixed to the center of mass of the aircraft structure (body frame). The vehicle dynamics in the body frame is described by (Roskam (1982))

$$m \dot{V}^b + m \Omega \times V^b = F^b$$

where $m$ represents the vehicle mass, $V^b = [u \ v \ w]^T$ denotes the linear velocity of the vehicle center of mass expressed in the body axis frame, $\Omega = [p \ q \ r]^T$ denote the angular velocity of the body frame, $J$ is the vehicle inertia matrix$^1$, $F^b$ represents the external applied forces expressed in the body frame and $M^b$ represents the external applied moments expressed in the body frame. In order to perform trajectory tracking the translational vehicle dynamics (1) must be expressed in earth axis coordinates. Considering the Euler yaw-pitch-roll rotation sequence. The rotation matrix that describes the orientation of the body reference frame relative to the earth frame is given by

$$R = \begin{bmatrix} c_\phi c_\theta & c_\phi s_\psi c_\theta & -s_\phi c_\psi & -s_\phi s_\theta \\ c_\phi s_\theta & -s_\phi c_\psi & c_\phi c_\psi & c_\phi s_\theta \\ c_\psi s_\theta + s_\phi c_\phi s_\psi & s_\phi c_\psi & c_\psi s_\theta & -c_\phi c_\psi \\ c_\psi s_\theta - s_\phi c_\phi s_\psi & -s_\phi c_\psi & c_\psi s_\theta & c_\phi c_\psi \end{bmatrix}$$

where $\Phi = [\phi \ \theta \ \psi]^T$ are the Euler angles with $\phi$ the roll angular displacement, $\theta$ the pitch angular displacement, $\psi$ the yaw angular displacement, and $c_x = \cos(x)$ and

$$sx = \sin(x).$$

Moreover, $\Omega$ is related to the Euler angles velocity as follows (Roskam (1982))

$$\dot{\Phi} = W(\Phi)^{-1} \Omega$$

with $t_s = \tan(x)$. From equation (3) we have that the relationship between the velocity components in the earth frame and the velocity components in the body frame is defined as

$$V^b = R V^e$$

where $V^e = [\dot{x} \ \dot{y} \ \dot{z}]^T$ is the linear velocity of the vehicle center of mass expressed in the earth frame. Thus, the translational vehicle dynamics expressed in the earth frame is described by

$$m \dot{V}^e = R^T F_e^b$$

The external applied forces expressed in the body frame are the vehicle weight and the total thrust produced by the four rotors, that is,

$$F_e^b = \begin{bmatrix} -mg s_\theta & 0 \\ mg c_\phi s_\theta & 0 \\ mg c_\phi c_\theta & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

where $T_i = \sum_{i=1}^4 T_i$ with $T_i$ the thrust of each rotor. The external applied moments in the body frame are defined as follows. Pitch motion is driven by the moment around $y^b$ produced by increasing the thrust of rotor 1 and reducing the thrust of rotor 3. Roll motion is generated in a similar way, that is, by producing a differential thrust between rotors 2 and 4. Due to the torque applied to the rotor shaft by the rotors a reactive torque of the same magnitude but opposite direction is experienced on the structure of the vehicle. To drive yaw motion a pair of rotors rotates in opposite direction to the other pair of rotors. Hence, the external moments applied to airframe can be defined as follows

$$M_e^b = \begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} (T_1 - T_3) \ell \\ (T_2 - T_4) \ell \\ Q_1 - Q_2 + Q_3 - Q_4 \end{bmatrix}$$

where $\ell$ is the distance between the rotor rotation axis and the aircraft center of mass and $Q_i$ is the reactive moment produced by rotor $i$.

It is shown in (A. Gessow (1978)) that the thrust generated by each rotor and the reactive moment can be expressed as

$$T_i = C_{T_i} \pi \tilde{r}_i^4 \rho \omega_i^2$$

$$Q_i = C_{Q_i} \pi \tilde{r}_i^2 \rho \omega_i^2$$

where $C_{T_i}$ is the rotor $i$ thrust coefficient, $\rho$ is the air density, $\tilde{r}_i$ is the radius of rotor $i$ and $\omega_i$ is the angular velocity of rotor $i$. Note that

$$Q_i = \frac{C_{Q_i}}{C_{T_i}} \tilde{r}_i T_i$$

thus, defining $\kappa_i = \tilde{r}_i^4 \rho$ and assuming that all rotors have the same characteristics we have that

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1 As the vehicle has two symmetry axes $J = \text{diag}(J_{xx}, J_{yy}, J_{zz})$. 

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Fig. 1. Rotary wing vehicle
with
\[
H = \begin{bmatrix}
C_{TK} & C_{TK} & C_{TK} & C_{TK} \\
-\ell C_{TK} & 0 & \ell C_{TK} & 0 \\
0 & \ell C_{TK} & 0 & -\ell C_{TK} \\
C_{QR} & C_{QR} & C_{QR} & \ell C_{QR}
\end{bmatrix}
\]

It is a common practice to use brushless motors to power the propeller. These brushless motors are commanded by electronic speed controllers driven by pulse width modulated signals (PWM) which are the actual control signal. Therefore it is necessary to identify a function relating the PWM signal with the angular velocity of rotors.

In what follows we will implicitly assume that the state signals the 0
have divided this task in three parts. First, the altitude

3.1 Altitude control

Equation (6) can be written as follows
\[
\begin{bmatrix}
T_x \\
L \\
M \\
N
\end{bmatrix} = H \begin{bmatrix}
E_x \\
E_y \\
E_z \\
E_w
\end{bmatrix}
\]

where ($x,y,z$) are the position coordinates in $0x^e y^e z^e$, is constrained to the set
\[
\mathcal{D} = \{\mathbb{R}^3 \times \mathbb{R}^3 \times \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \times \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \times \left(-\pi, \pi\right) \times \mathbb{R}^3\}
\]
in what follows we will implicitly assume that the state remains in $\mathcal{D}$ for all $t$.

The control objective is to asymptotically track a pre
scribed path for the vehicle Cartesian position ($x,y$), a
desired vehicle altitude $z$ and a desired yaw angle $\psi$.

In what follows we show that the considered control objective is achievable under the following standing assumption
\[
A1 \text{ All system states are measurable and all system physical parameters are known.}
\]

3. CONTROL DESIGN

This Section describes the control design procedure. We have divided this task in three parts. First, the altitude controller is designed, then by means of virtual control signals the $0x^e y^e z^e$ plane controller is developed and finally, the altitude controller is designed to produce the necessary virtual control signals.

3.1 Altitude control

Equation (6) can be written as follows
\[
\begin{align*}
\dot{x} &= -T_x (c_\phi s_\psi + s_\phi s_\psi) \\
\dot{y} &= -T_x (c_\phi s_\psi - s_\phi c_\psi) \\
\dot{z} &= -T_c c_\phi + m g
\end{align*}
\]

Defining
\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2
\end{bmatrix} = \begin{bmatrix}
z - z_d \\
\dot{z} - \dot{z}_d
\end{bmatrix}
\]
the third equation in (9) can be written as
\[
\dot{z}_1 = z_2 \\
\dot{z}_2 = -c_\phi c_\psi \frac{T_c}{m} + g - \dot{z}_d
\]

Vertical motion control can be obtained by defining
\[
T_c = \frac{m (g - \gamma_z)}{c_\theta c_\phi}
\]
where $\gamma_z$ is a function of $Z$ and time such that the dynamic system
\[
\begin{align*}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= \gamma_z - \dot{z}_d
\end{align*}
\]
is asymptotically stable.

3.2 0x^e y^e plane control

Replacing (11) into the first two equations of (9) we obtain
\[
\begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} = -(g - \gamma_z) \begin{bmatrix}
c_\psi & s_\psi \\
-s_\psi & c_\psi
\end{bmatrix} \begin{bmatrix}
\dot{\theta}_d & \dot{\phi}_d \\
\dot{\phi}_d & \dot{\theta}_d
\end{bmatrix} \begin{bmatrix}
t_\phi \\
t_\theta
\end{bmatrix}
\]

In terms of the path following error coordinates
\[
X_1 = \begin{bmatrix}
x \\
y
\end{bmatrix} - X_d(\eta_1),
\]
\[
X_2 = \begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} - \nabla_{\eta_1} X_d(\eta_1) \dot{\eta}_1
\]

the dynamic equations (13) read as
\[
\begin{align*}
\dot{X}_1 &= \dot{x} - X_d(\eta_1), \\
\dot{X}_2 &= -(g - \gamma_z) \begin{bmatrix}
c_\psi & s_\psi \\
-s_\psi & c_\psi
\end{bmatrix} \begin{bmatrix}
\dot{\theta}_d & \dot{\phi}_d \\
\dot{\phi}_d & \dot{\theta}_d
\end{bmatrix} \begin{bmatrix}
t_\phi \\
t_\theta
\end{bmatrix}
\end{align*}
\]

where we have added $\dot{\theta}_d$ and $\dot{\phi}_d$ as the virtual control inputs and taken into account that
\[
\dot{\eta}_1 = v_s - \eta_2
\]

with $v_s$ the desired velocity on the path. Defining
\[
\begin{align*}
\phi_d &= \arctan \left( \frac{a_1}{a_3} \right) \\
\theta_d &= \arctan \left( \frac{a_2}{a_3} \right)
\end{align*}
\]

where $a_1 = c_\phi (s_\psi \gamma_x - c_\psi \gamma_y), a_2 = c_\psi \gamma_x + s_\psi \gamma_y, a_3 = g - \gamma_z$ and
\[
\begin{bmatrix}
\gamma_x \\
\gamma_y
\end{bmatrix} = K_{Px} X_1 + K_{Dx} X_2 - \nabla_{\eta_1} X_d(\eta_1) \begin{bmatrix}
v_s \\
\eta_2
\end{bmatrix}
\]

with $K_{Px}$ and $K_{Dx}$ positive definite matrices. Substituting (16) into (14) we obtain
\[
\begin{align*}
\dot{X}_1 &= X_2 \\
\dot{X}_2 &= -K_{Px} X_1 - K_{Dx} X_2 \\
&\quad - a_3 \begin{bmatrix}
c_\psi & s_\psi \\
-s_\psi & c_\psi
\end{bmatrix} \begin{bmatrix}
\dot{\theta}_d & \dot{\phi}_d \\
\dot{\phi}_d & \dot{\theta}_d
\end{bmatrix} \begin{bmatrix}
t_\phi \\
t_\theta
\end{bmatrix}
\end{align*}
\]

Notice that a corner stone in the controller design for the
$0x^e y^e$ plane dynamics is the fact that the signal $(g - \gamma_z)$...
needs to be bounded away from zero. To achieve this we propose
\[ \gamma_z = \tilde{z}_d - \frac{\epsilon_z}{2} \left[ \tanh \left( \frac{2\lambda_2}{\epsilon_z} \tilde{z}_1 \right) + \frac{1}{2} \tanh \left( \frac{2\lambda_2}{\epsilon_z} \tilde{z}_2 \right) \right] \]
which as stated in (Kaliora and Astolfi (2004)) achieves local exponential stability of (12), for adequately selected positive constants \( \lambda_1, \lambda_2 \) and \( \epsilon_z \), and bounds away from zero the term \( (g - \gamma_z) \), provided \( |\tilde{z}_d| \) is bounded, since
\[ |\gamma_z| \leq |\tilde{z}_d| + \frac{3}{4}\epsilon_z = \kappa_z \]
Note that the closed-loop dynamics (18)-(17) can be written as follows
\[ \dot{X} = AX - a_d G + F\hat{\eta}_2 \]
where
\[ X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, G = \begin{bmatrix} 0 \\ \begin{bmatrix} 0 & s_\psi \\ c_\psi & 0 \end{bmatrix} \end{bmatrix}, A = \begin{bmatrix} 0_2 & I_2 \\ -K_{P\epsilon} - K_{D\kappa} \end{bmatrix}, F = \nabla_{\gamma_1}X_d(\eta_1) \]
with \( 0_2 \) and \( I_2 \) the zero matrix and the identity matrix of dimension \( 2 \times 2 \).
Consider now the Lyapunov function
\[ V_X = X^T P X + \frac{1}{2} \gamma_2^2, \quad \gamma > 0 \]
whose time derivative along the trajectories of (19) is given by
\[ \dot{V}_X = X^T (PA + A^T P) X - 2\alpha_3 X^T PG \\
+ (2X^T PF + \gamma_2 \hat{\eta}_2) \]
defining
\[ \hat{\eta}_2 = -(2X^T PF + \gamma_2) \]
and selecting the gains \( K_{P\epsilon} \) and \( K_{D\kappa} \) in such a way that \( A \) is Hurwitz we obtain
\[ V_X = X^T QX - 2\alpha_3 X^T PG \\
- (X^T PF + \gamma_2 \hat{\eta}_2)^2 \]
for positive definite matrices \( P \) and \( Q \).
At this point it is not possible to conclude about the stability properties of the vehicle’s Cartesian dynamics because of the term \( G \). In the next section the attitude dynamics will be commanded to track the angles defined in (16). This will allow us to establish the stability properties of the vehicle’s dynamics.

3.3 Attitude control

It is well known that there exists a passive map between the control inputs and the rigid body angular velocity in the attitude dynamics. This fact has been exploited to design passivity based schemes to control the attitude dynamics (Lizarralde and Wen (1996)) even with bounded controllers (J. F. Guerrero-Castellanos and Lesecq (2012)). Here we will also exploit the passivity properties of the attitude dynamics.

Define the following error signals
\[ \hat{\Phi} = \Phi - \Phi_d \]
\[ \hat{\Omega} = \Omega - \Omega_d \]
Expressing the attitude kinematics (4) and attitude dynamics (2) in terms of the error signals (21) one has
\[ \dot{\hat{\Phi}} = W(\Phi)^{-1}\hat{\Omega} + W(\Phi)^{-1}\Omega_d - \dot{\Phi}_d \]
\[ \dot{\hat{\Omega}} = S(J\hat{\Omega})\hat{\Omega} + S(J\hat{\Omega})\Omega_d + \hat{M}^b \]
where we have defined \( S(\cdot) \) as the matrix that performs the vector’s cross product, that is, \( d_1 \times d_2 = S(d_1)d_2 \), we have considered that the following equation holds
\[ J\hat{\Omega}_d = S(J\Omega_d)\hat{\Omega}_d + M^b_d \]
and finally we have defined
\[ \hat{M}^b = M^b - M^b_d \]
Given (16) we have
\[ \hat{\Phi}_d = \Phi_1(X, \eta, Z, \Phi) + \Phi_2(X, Z, \eta, \Phi)W(\Phi)^{-1}\Omega \]
with
\[ \eta = (\eta_1 \eta_2)^T \]
\[ \Phi_1 = \begin{bmatrix} c_\theta a_3 (s_\phi \gamma_x - c_\psi \gamma_y) + a_1 \gamma_z \\ a_1^2 + a_2^2 \end{bmatrix}, \Phi_2 = \begin{bmatrix} 0 -s_\theta (s_\phi \gamma_x + c_\psi \gamma_y) c_\psi (s_\phi \gamma_x - s_\psi \gamma_y) \\ 0 0 -s_\psi \gamma_x - c_\psi \gamma_y \end{bmatrix} \]
and
\[ a = a_3 \left( 1 + \frac{c_\phi^2 (s_\psi \gamma_x + c_\psi \gamma_y)^2}{a_3^2} \right) \]
We define the attitude dynamics controller as follows
\[ \Omega_d = W(\Phi) \left( -K_P \hat{\Phi} + \Phi_1 + \Phi_2 W(\Phi)^{-1}\Omega \right) \]
\[ \hat{M}^b = -S(J\hat{\Omega})\hat{\Omega}_d - K_D \hat{\Omega} - W(\Phi)^{-T} \hat{\Phi}_c \]
with \( K_P \) and \( K_P \) positive definite matrices.
Observe that to implement the attitude controller defined in (25) it is necessary to compute \( \hat{\Omega}_d \) in order to obtain \( M^b_d \) from (23). However, since \( \hat{\Phi} \) and \( \eta \hat{d} \) depend on the pitch and yaw angles, and these angles have a relative degree equal 2, the first time derivative of \( \Omega_d \) has the following form
\[ \dot{\hat{\Omega}}_d = \Phi_3(\Phi, \Omega, X, \eta, Z) \]
\[ + W(\Phi) \Phi_2 W(\Phi)^{-1}\Omega \]
therefore, \( M^b_d \) is computed from (24) as follows
\[ M^b_d = \Phi_4^{-1} \left[ -S(J\Omega_d)\Omega_d + J\Phi_3 - S(J\hat{\Omega})\hat{\Omega}_d \right. \]
\[ \left. -K_P \hat{\Omega} \right] \]
with \( I \) the \( 3 \times 3 \) identity matrix and
\[ \Phi_4 = [I + JW\Phi_2 W^{-1}J^{-1}] \]
It is easy to verify that
\[ \det(\Phi_4) = 1 \]
The properties of the proposed controller can be stated as follows:
Proposition 1. Under assumption A1. Consider the rotary wing vehicle dynamics described by equations (1)-(2) in closed–loop with the controller defined by equations (11), (17), (20) and (26). Assume that the desired path 𝑋𝑑(𝜂) is bounded and has bounded derivatives with respect to 𝜂. Then, there exist positive constants 𝜆2, 𝜆2, 𝜎 and 𝜂2 and positive definite matrices 𝐾𝑝, 𝐾𝑑, 𝐾 and 𝐾 diarr such that the closed–loop dynamics is locally asymptotically stable in 𝒫.

Proof. The closed–loop dynamics (1)-(2)-(11)-(17)-(20)-(26) expressed in terms of the error coordinates reads as

\[ \dot{X} = AX - a_3 G - F(2X^TPF + \gamma \eta) \]

\[ \dot{\eta}_1 = \eta_2 \]

\[ \dot{\eta}_2 = -(2X^TPF + \gamma \eta) \]

\[ \dot{z}_1 = z_2 \]

\[ \dot{z}_2 = -\frac{\epsilon_2}{2} \left( \tanh \left( \frac{2\lambda z_1}{\epsilon_2} \right) + \frac{1}{2} \tanh \left( \frac{2\lambda z_2}{\epsilon_2} \right) \right) \] (28)

\[ \begin{bmatrix} \dot{\Phi} \\ J\dot{\Omega} \end{bmatrix} = \begin{bmatrix} -K_P & W(\Phi)^{-1} \\ -W(\Phi)^{-T} S(J\Omega) - K_D \end{bmatrix} \begin{bmatrix} \dot{\Phi} \\ \dot{\Omega} \end{bmatrix} \]

From (Kaliora and Astolfi (2004)) we know that the vertical closed-loop dynamics, the fourth and fifth equations in (28) are locally exponentially stable, thus there exists a Lyapunov function

\[ V_z = \frac{1}{2} Z^TPZ \]

such that locally

\[ \dot{V}_z = -Z^TPZ \]

holds. Consider now, the Lyapunov function

\[ V = V_X + \frac{1}{2} \Phi^T \Phi + \frac{1}{2} \Omega^T J\Omega + V_z \]

whose time derivative along the trajectories of (28) is given by

\[ \dot{V} = -X^T Q X - 2a_3 X^T P G - (X^TPF + \gamma \eta z_2)^2 - Z^T Z - \Phi^T K_P \Phi - \Omega^T K_D \Omega \]

Considering that

\[ |G| \leq \kappa|\Phi| \] (29)

the time derivative of the Lyapunov function can be upper bounded as

\[ \dot{V} \leq -\lambda_{\min}(Q)|X|^2 + 2\kappa_2 \lambda_{\max}(P)|X||\Phi| \]

\[ -\lambda_{\min}(K_P)|\Phi|^2 - \lambda_{\min}(K_D)|\Omega|^2 \]

\[ -|X^TPF + \eta z|^2 - |Z|^2 \]

From the Young’s inequality we have

\[ \dot{V} \leq -\left[ \lambda_{\min}(Q) - \kappa_2\sigma \lambda_{\max}(P) \right]|X|^2 \]

\[ -\left[ \lambda_{\min}(K_P) - \frac{1}{\sigma} \lambda_{\max}(P) \kappa_2 \sigma \right]|\Phi|^2 - \lambda_{\min}(K_D)|\Omega|^2 \]

\[ -|X^TPF + \eta z|^2 - |Z|^2 \]

for positive constant \( \sigma > 0 \). Hence, by selecting the gains in such a way that

\[ \lambda_{\min}(Q) - \sigma \lambda_{\max}(P) \kappa_2 \sigma > 0 \]

\[ \lambda_{\min}(K_P) - \frac{1}{\sigma} \lambda_{\max}(P) \kappa_2 \sigma > 0 \] (30)

we have the proof is completed.

4. NUMERICAL SIMULATIONS

Numerical simulations were carried out to assess the performance of the controller proposed. The numerical value of the vehicle parameters are presented in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>m (kg)</td>
<td>1.120</td>
<td>C_Q</td>
<td>0.1</td>
</tr>
<tr>
<td>g (m/s²)</td>
<td>9.81</td>
<td>ℓ (m)</td>
<td>0.214</td>
</tr>
<tr>
<td>r (m)</td>
<td>0.127</td>
<td>Izz (kgm²)</td>
<td>0.002</td>
</tr>
<tr>
<td>ρ (kg/m³)</td>
<td>1</td>
<td>Jyy (kgm²)</td>
<td>0.002</td>
</tr>
<tr>
<td>C_T</td>
<td>0.5</td>
<td>Izz (kgm²)</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Table 1. Rotary wing vehicle parameters

The desired trajectory is defined as follows

\[ x_d = \sin(\eta) \]

\[ y_d = \cos(\eta) \]

\[ z_d = -2 + \sin(t) \]

\[ \psi_d = 0 \] (31)

We consider the following initial conditions \((x, y, z, \phi, \theta, \psi) = (0, 0, 0, 0, 0, 0)\). The controller parameters are summarized in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ</td>
<td>50</td>
</tr>
<tr>
<td>ℓ</td>
<td>10</td>
</tr>
<tr>
<td>Izz</td>
<td>diag{1,1.5}</td>
</tr>
<tr>
<td>K_P</td>
<td>diag{0.85,0.75}</td>
</tr>
<tr>
<td>K_D</td>
<td>diag{1,1.5,0.5}</td>
</tr>
<tr>
<td>K_D</td>
<td>diag{0.85,0.75,0.4}</td>
</tr>
</tbody>
</table>

Table 2. Controller parameters

The path error signals are shown in Figure 2 The time history of the control signals is shown in Figure 3.

We consider now the case of non zero localization errors. We consider the following model for the position error measurement

\[ \delta_P(t) = \Delta_1 \] (32)

where \( \Delta_1 \) represents the random error. Figure 4 shows the time histories of the path following errors as it can be observed the proposed controller is not able to drive it to zero, however the path errors have a bounded behavior. Figure 5 shows the time histories of the control inputs.
5. CONCLUSION

The problem of path following for the quadrotor mobile robot has been addressed and solved by means of a nonlinear controller based on a backstepping inspired controller and an output maneuvering controller, as introduced in (R. Skjetne and Kokotovic (2004)). We have analyzed the proposed controller in two scenarios, the first one assumes that the quadrotor mobile robot Cartesian position is accurately measured or estimated and the second case considers that there is an error in the quadrotor Cartesian localization system. As expected the performance of the proposed controller is degraded due to the errors in the localization system however it keeps all signals bounded. Numerical simulations have been used to illustrate the properties of the closed-loop system.

Some issues are left open in this work. First, the analysis of the stability in the case that the localization systems is not accurate. Second, the experimental test of the proposed controller and the inclusion of wind.

REFERENCES


